Separating Computational and Statistical Differential Privacy in the Client-Server Model

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Overview

- Differential Privacy (DP)
- Computational Differential Privacy (CDP)
- Previous Work & Main Contributions
- Sketch Result: Separation of CDP and DP
- Conclusion

Differential Privacy (Client-Server Setting)

Database: D

Name	Age	Height	Smoke
Alice	13	147	Y
Charlie	27	176	N
$\vdots$	$\vdots$	$\vdots$	$\vdots$
Eve	42	173	Y

Analyst

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distribution of $M(D) \approx$ distribution of $M(D')$

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M is (ε, δ)-differentially private if $\forall\ D \sim D'$ and output set T,

$$\boxed{\Pr[M(D) \in T] \leq e^{\varepsilon} \Pr[M(D') \in T] + \delta}$$

[Dwork, McSherry, Nissim, Smith '06]

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$1 + \varepsilon$ (ε: small constant), (δ: negligible)

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Privacy vs. Utility

Differential Privacy Results

Algorithms:

- Histogram [DMNS06]
- Exponential Mechanism [MT07]
- Synthetic Data [BLR08]
- Private Multiplicative Weights [HR10]
- Boosting [DRV10]
- Private Learning [KLNRS08]
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Lower Bound Results:

- Reconstruction Attack [DN03]
- Geometric Argument [HT10]
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Q: Can we obtain improved algorithms
by relaxing the definition?

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Def M is ε-SIM-CDP if $\exists\ (\varepsilon, \mathrm{negl})$-DP M' s.t. $\forall\ D$

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$$\boxed{\text{DP} \subseteq \text{SIM-CDP} \subseteq \text{IND-CDP}}$$

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Goal Computing a joint function of private datasets.

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$\boxed{\text{DP} \subsetneq \text{SIM-CDP in the case of two or more parties.}}$

2-party task Hamming distance

n-party task Sum of n bits

DP Error: $\Theta(\sqrt{n})$ [BNO08, MPRV09, MMPRTV10, CSS12]

CDP Error: $O(1)$ (using MPC) [DKMMN06, BNO08]

Our Focus: Client-Server Setting

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- Error $= L_p$ norm on $\mathbb{R}^{O(1)} \Rightarrow$ can convert IND-CDP to DP with $1/\operatorname{poly}(k)$ additive increase in error.
- Cannot separate IND-CDP and DP with black-box 'generic' crypto primitives (e.g. OWF, TDP).

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Our Results

Thm1 (DP $\neq$ SIM-CDP) Assume NIZKs for NP & sub-exponentially secure OWF.

Then $\exists$ poly-time computable utility function $U(D, M(D))$ s.t.

1. $\exists$ poly-time SIM-CDP mechanism M^{CDP} s.t.
$$\forall D, \Pr[U(D, M^{\mathrm{CDP}}(D)) = 1] \geq 1 - \operatorname{negl}(k).$$
2. $\forall$ poly-time DP mechanism $M^{\mathrm{DP}}, \exists D$ s.t.
$$\Pr[U(D, M^{\mathrm{DP}}(D)) = 1] \leq \operatorname{negl}(k).$$

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Thm2 (Extension of [GKY11]) Error $=$ metric with $O(\log k)$ doubling dimension $\Rightarrow$ can convert IND-CDP to DP with $O(1)$ multiplicative increase in error.

Proof Outline

- Tools
  - "Exponentially Extractable" Zaps [Dwork, Naor '07]. (Based on NIZK)
  - Sub-exponentially Strongly Unforgeable Digital Signature Scheme (Based on sub-exponentially secure OWF)

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- Define Task: zap proof of existence of a signature.
- Claims
  1. $\exists$ a (non-efficient) DP mechanism with high utility.
  2. $\exists$ an efficient SIM-CDP with high utility. ($\stackrel{c}{\approx}$ to the DP mechanism)
  3. No efficient DP mechanism with non-negligible utility (Otherwise break the signature scheme).

Zaps (2-message public coin witness indistinguishability)

$L \in \mathbf{NP}$ witness relation: $(x, w) \in R_L$ security parameter: k

Verifier		Prover
x	$\xrightarrow{\rho \leftarrow \{0,1\}^\ell}$	x, w
Verifier V	$\xleftarrow{\pi = P(1^k, x, w, \rho)}$	Prover P

$V(1^k, x, \rho, \pi) = 0$ or 1

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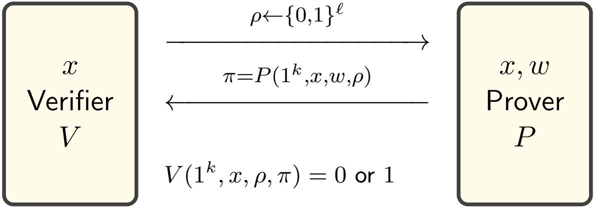

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- *Soundness*
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- *Completeness*
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- *Witness Indistinguishability* (vs. adversarial V^*)
- *Extractability*: Algorithm E running in time $2^{O(k)}$ s.t. $\forall x$

$$(x, \rho, \pi) \longrightarrow \boxed{\text{Extractor } E} \longrightarrow w = E(1^k, x, \rho, \pi)$$

The Task

- $(\mathrm{Gen}, \mathrm{Sign}, \mathrm{Ver})$: Sub-exponentially secure signature scheme.
- $(P_{\mathrm{zap}}, V_{\mathrm{zap}}, E_{\mathrm{zap}})$: Exponentially extractable zap.

The Task

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Dataset D $\rightarrow$ Mechanism M $\rightarrow$ $M(D) = \pi$: Zap proof that "$\exists(m, \sigma)$ s.t. $\mathrm{Ver}_{vk}(m, \sigma) = 1$"

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Dataset D → Mechanism M → $M(D) = \pi$: Zap proof that "$\exists (m, \sigma)$ s.t. $\mathrm{Ver}_{vk}(m, \sigma) = 1$"

- Utility: $U(D, M(D)) = 0$ or 1.
  - If $> 90\%$ rows are of the form $(\hat{vk}, \hat{\rho}, m_i, \sigma_i)$ where $\mathrm{Ver}_{\hat{vk}}(m_i, \sigma_i) = 1$, then output $V_{\mathrm{zap}}(\hat{vk}, \hat{\rho}, \pi)$.
  - Otherwise, output 1.

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   Idea If there exists such an M, combine M and $2^{O(k_{\text{zap}})}$-time zap extractor to construct a $2^{O(k_{\text{zap}})}$-time forger for digital signature.

   Violate Sub-exponential-security of digital signature. (complexity leveraging [CGGM00])

Conclusion

- DP $\subsetneq$ SIM-CDP $\subseteq$ IND-CDP (in client-server model)
  Assuming sub-exponential OWF and NIZK, we construct a task s.t.
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